NetSci-X 2015 – International School and Conference on Network Science – Rio, Brazil, January 14-16, 2015

Time Centrality

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INTRODUCTION

The concept of centrality in graph theory and network analysis usually refers to metrics that assess the relative importance of vertices (nodes) within a graph (network). Different ways of measuring node centrality have been proposed for decades, each of them suited to assess node centrality from a different point of view. We argue that, for some applications modelled in **Time- Varying Graph (TVG)** scenarios, more important than identifying the central nodes under a given definition, identifying the key time instants for taking certain actions can be of upmost importance for a more effective outcome. For instance, it can be interesting to determine the best time period at which a vaccine should be applied in a population to achieve better responses to an epidemic spreading or the best time period to start a faster information diffusion at lower costs.

MODEL FOR REPRESENTING TVGs

We have used the model proposed by Wehmuth *et al.* [1], which unifies previous TVG models in literature. The model represents a TVG as an object H = (V, E, T), where V is the set of nodes, T is the finite set of time instants, and E ⊆ V x T x V x T is the set of edges. A dynamic edge *e* in a TVG H is defined as an ordered quadruple *e* = (u, t_a, v, t_b), where u, v ∈ V(H) are the origin and destination nodes, while t_a, t_b ∈ T(H) are the origin and destination time instants, respectively. A dynamic edge *e* = (u, t_a, v, t_b) should be understood as a connection from node u at time t_a to node v at time t_b. In Figure 1, we show the adjacency matrix form of an example TVG G1. Each one of the four nodes (identified as 0, 1, 2, and 3) of G1 clearly appears as a separate entity in each of the three time instants (t_0, t_1, and t_2).

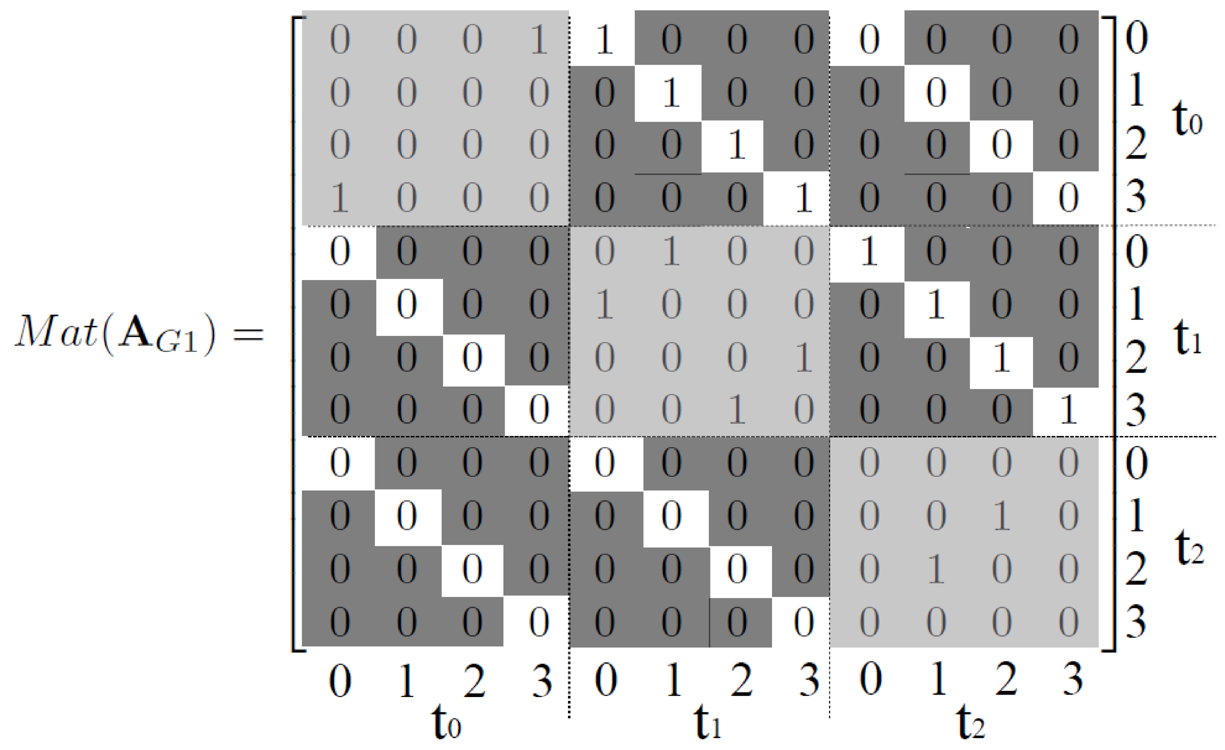
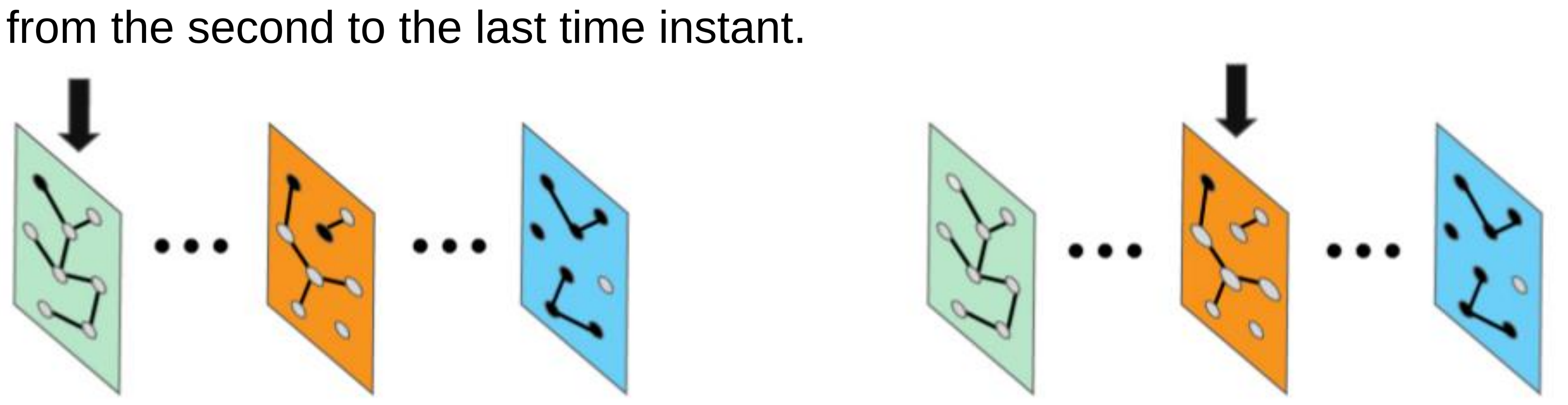


Figure 1 - The main block diagonal (lightly shaded) contains the entries corresponding to the spatial edges at each time instant. The unshaded entries at the off-diagonal blocks correspond to the temporal edges. The dark shaded entries are the ones that correspond to the mixed edges.

TIME CENTRALITY

We define **time centrality** as the measure of the relative importance of a given time instant compared with other time instants of the same TVG. One simple way to represent a TVG is to use a sequence of snapshots that represent the graph structure at a given time instant. Consider Figures 2(a) and 2(b) that represent the same snapshot sequence of an arbitrary TVG. In each snapshot, black nodes received information and grey nodes did not. In Figure 2(a), diffusion starts in the earliest time instant (green snapshot), reaches two nodes in the intermediate time instant (orange snapshot) and reaches almost all nodes in the last time instant (blue snapshot). In Figure 2(b), diffusion starts only in the intermediate time instant (orange snapshot) and reaches the same number of nodes in the last time instant (blue snapshot). It can be seen from this simple example that the diffusion was more efficient in the latter case, since the number of infected nodes is the same as in the former case, whereas the diffusion lasted just one time instant, from the second to the last time instant.



(a) Diffusion starts in the green snapshot (b) Diffusion starts in the orange snapshot

Figure 2 - Example of time centrality where diffusion starting in different time instants change the efficiency of diffusion.

PROPOSED METRICS

We have proposed *three time centrality metrics*. **Time degree centrality** of t_a is given by the sum of the degrees of temporal nodes (u, t_a), where u refers to any node belonging to V(G). **Cover time centrality** measures the time required for an information to reach a given percentage or number of nodes in a TVG from an initial time instant by diffusion. **Coverage centrality** computes the amount (or percentage) of TVG nodes reached by information diffusion, given a number of fixed steps in Breadth-First Search algorithm.

RESULTS

We have analyzed a face-to-face contact network, representing all contacts among people in a hospital ward for a period of time [2]. The studied metrics have been found the most important time instants in TVG.

- Time Degree

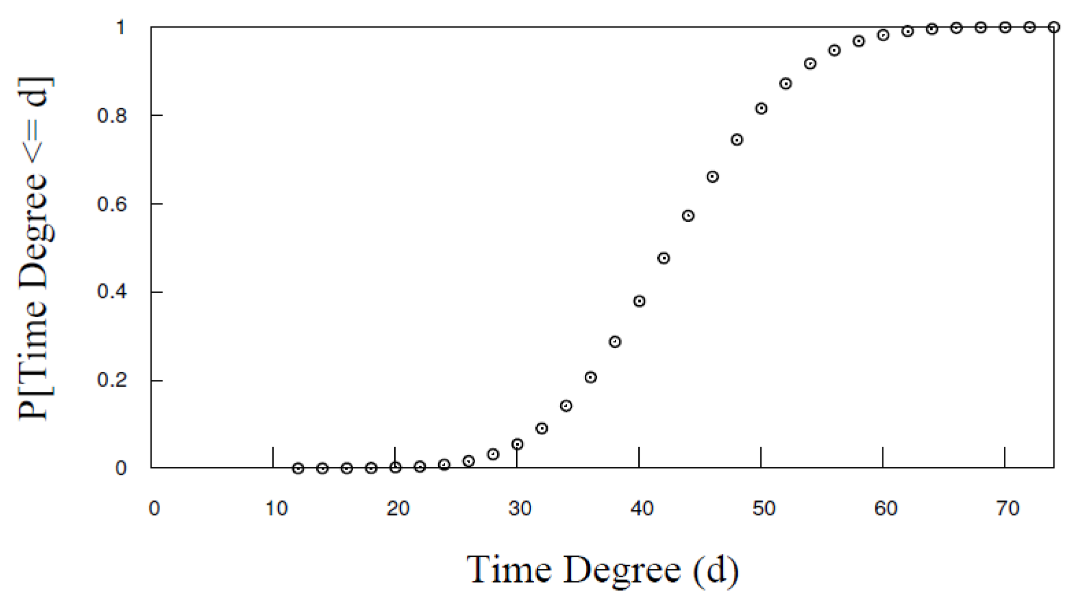


Figure 3 - Time degree CDF. The average of the top 20% time instants is 30% higher than the average of all time instants.

- Time Degree Variation

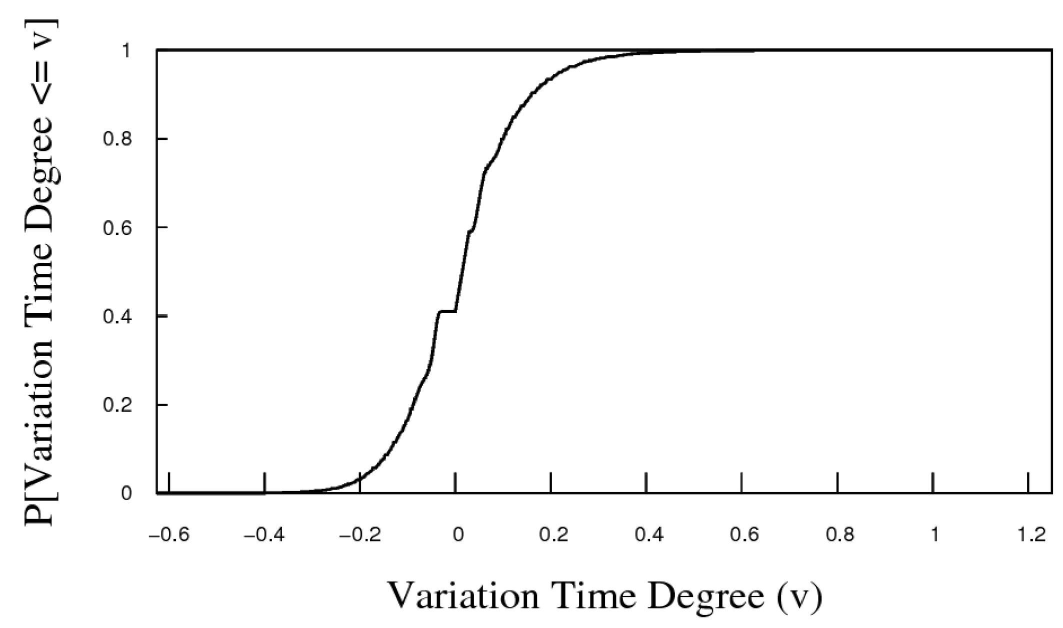


Figure 4 - Time degree Variation CDF. The average of the top 20% time instants is 0.184, whereas for all time instants the average is 0.007.

- Cover Time

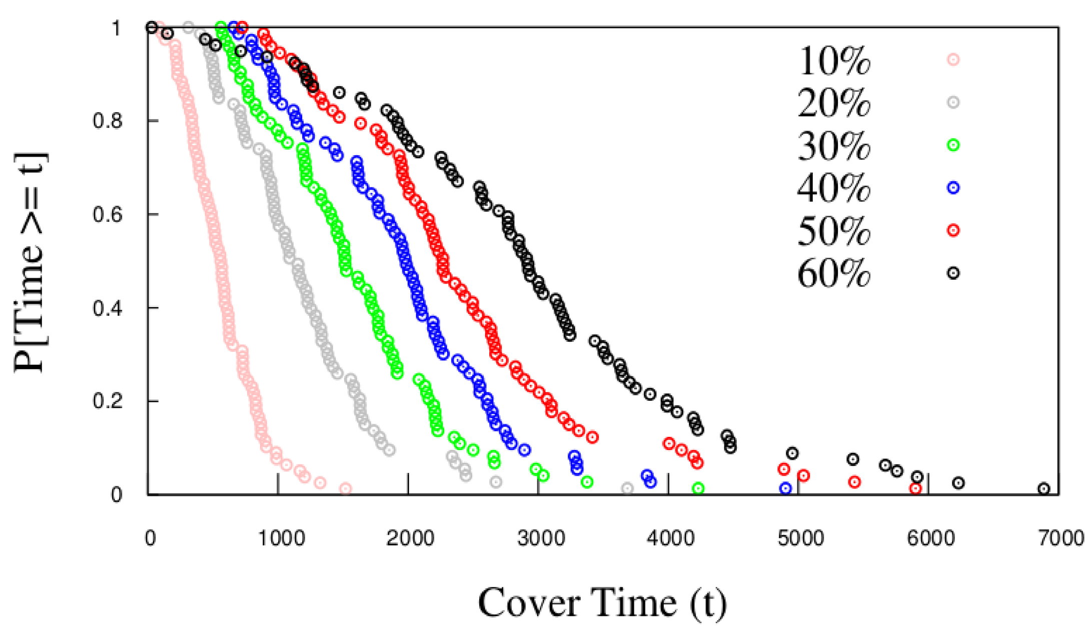


Figure 5 - Cover Time CDF for different maximum network sizes. The average of the top 20% time instants is 30% lower than the average of all time instants for 60% network size and 40% lower for 30% network size.

- Coverage Centrality

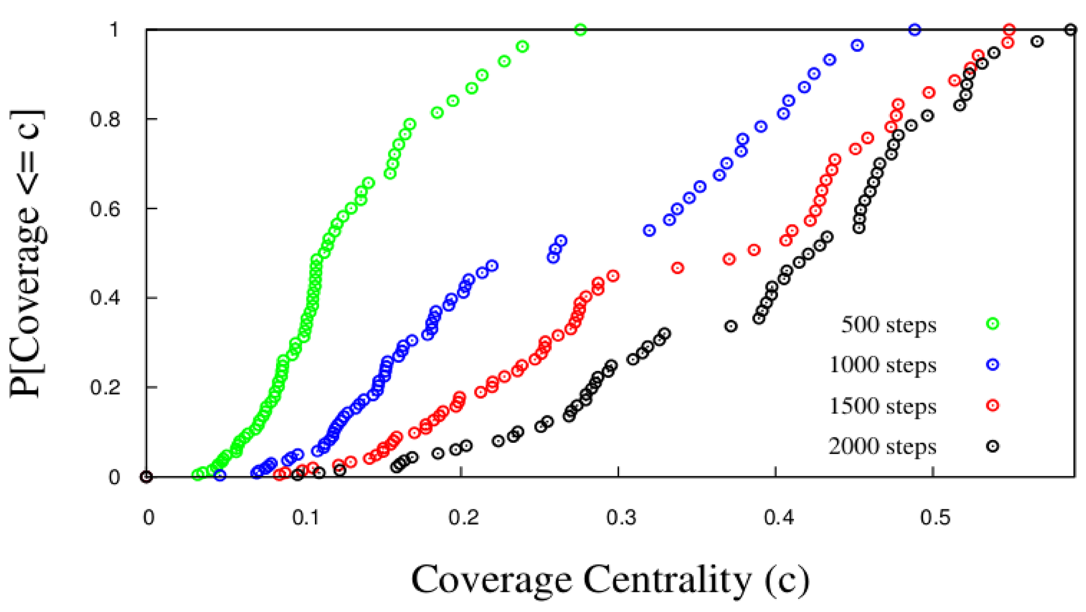


Figure 6 - Coverage centrality CDF for different number of fixed steps. The average of the top 20% time instants is 42% higher than the average of all time instants for 2000 steps diffusion, and 65% higher for 1000 steps diffusion.

References

[1] WEHMUTH, K.; ZIVIANI, A.; FLEURY, E. **A Unifying Model for Representing Time-Varying Graphs**. CoRR, 2014.

[2] LUCET, J. C.; LAOUENAN, C.; CHELIUS, G.; VEZIRIS, N.; LEPELLETIER, D.; FRIGGERI, A.; ABITEBOUL, D.; BOUVET, E.; MENTRÉ, F.; FLEURY, E. **Electronic Sensors for Assessing Interactions between Healthcare Workers and Patients under Airborne Precautions**. PLoS ONE, Public Library of Science, 2012, 7 (5)