

Fast Centrality-Driven Diffusion in Dynamic Networks

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ABSTRACT

Diffusion processes in complex dynamic networks can arise, for instance, on data search, data routing, and information spreading. Therefore, understanding how to speed up the diffusion process is an important topic in the study of complex dynamic networks. In this paper, we shed light on how centrality measures and node dynamics coupled with simple diffusion models can help on accelerating the cover time in dynamic networks. Using data from systems with different characteristics, we show that if dynamics is disregarded, network cover time is highly underestimated. Moreover, using centrality accelerates the diffusion process over a different set of complex dynamic networks when compared with the random walk approach. For the best case, in order to cover 80% of nodes, fast centrality-driven diffusion reaches an improvement of 60%, i.e. when next-hop nodes are selected by using centrality measures. Additionally, we also propose and present the first results on how link prediction can help on speeding up the diffusion process in dynamic networks.

Categories and Subject Descriptors

C.2 [Computer-Communication Networks]: Network monitoring, Public networks

General Terms

Performance

Keywords

Complex Networks; Dynamic Networks; Centrality; Diffusion Processes

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WWW 2013 Companion, May 13–17, 2013, Rio de Janeiro, Brazil.
ACM 978-1-4503-2038-2/13/05.

1. INTRODUCTION

Complex networks currently pervade our daily lives [8, 9, 18]. Investigating the spread of information, or the diffusion process, in complex networks is key from both theoretical and applied perspectives [3, 14, 17, 23, 33]. Furthermore, the understanding of diffusion processes becomes even more challenging if considering dynamic complex networks [2] (e.g., the topology as well as the network elements may change over time). Dynamic systems arise in many different areas, for instance, from packet switching networks in the Internet to disease spreading on social networks [13].

There is a lot of research investigating diffusion processes in complex networks that relies on a static view of the network [1, 5, 7, 15, 22, 26]. Moreover, several works are interested in the proportion of nodes covered by the information spread, no matter the time needed to achieve this task [3, 11, 17, 22]. Some papers deal with the acceleration of diffusion processes, but their proposed algorithms typically use additional information, as explicit social relationships [14, 23]. As a consequence, most previously proposed diffusion algorithms either not necessarily achieve the performance they should or rely on complementary, but not necessarily available, information.

In this paper, we investigate the fast centrality-driven diffusion in complex dynamic networks. First, we show that the traditional models of diffusion process, which disregard system dynamics, underestimate the time information takes to reach all nodes (i.e., the cover time). For the analyzed network, the cover time in a static view of the network is underestimated in about 20 times compared with the time when real system dynamics is modeled. Second, results show that using centrality accelerates the diffusion process over complex dynamic networks with different characteristics. For instance, diffusion based on betweenness centrality needs only 40% of the time spent by a random walk algorithm to reach 80% of the network nodes. Third, we present results on how to improve the network cover time when link prediction is applied.

The remainder of this paper proceeds as follows. We introduce the adopted network model and centrality measures in Section 2. The diffusion algorithms we consider are presented in Section 3. Section 4 describes the

datasets we use. The impact of considering system dynamics and applying centrality measures is presented in Section 5. Section 6 discusses how link prediction can help the diffusion process. Related works are discussed in Section 7. Finally, Section 8 presents our concluding remarks and plans for future work.

2. NETWORK MODEL

Let a network be a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where \mathcal{V} is the set of nodes and \mathcal{E} is the set of links. The network dynamics, observed during T time units, is represented by a set of graphs $\mathcal{G}_t = \{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_n\}$. Each graph $\mathcal{G}_t = (\mathcal{V}_t, \mathcal{E}_t)$ is a snapshot from the network model during $1 \leq \delta \leq T$ time units, i.e., we use δ as a tuning factor to determine how many ($n = \lceil T/\delta \rceil$) snapshots we take into account in the analysis.

In short, large (low) values of δ provide a higher (smaller) level of information aggregation in each snapshot and a lower (higher) perception of the network dynamics. As pointed out in [13], the model presented here derives static graphs that capture both temporal and topological properties of the system, accumulating the contacts over some time to form edges at each snapshot.

In this paper, w.l.o.g., we assume that the set of nodes remains unchanged over time. As a consequence, there is no node join or departure and the network dynamics is restricted to topology dynamics, i.e., link changes. More precisely, \mathcal{G}_t is reduced to $\mathcal{G}_t = (\mathcal{V}, \mathcal{E}_t)$. This model may represent, during a certain period of interest, a large number of real networks, such as router-level networks, low churn P2P systems, (online) social networks, and so on.

Centrality metrics intend to determine the most important (central) nodes in a network [24]. There are many possible definitions of node importance, and correspondingly as many centrality metrics. Hence, we consider the following most common centrality measures found in literature:

1. *Degree* - The degree centrality d_v of node v is defined as the ratio between the node degree, $d(v)$, and the maximum node degree in the network:

$$d_v = \frac{d(v)}{\max_{v \in \mathcal{V}} d(v)}. \quad (1)$$

2. *Betweenness* - The betweenness β_v of node v is the fraction of shortest paths connecting all pairs of nodes that pass through v . In other words, let $\sigma_{(j,k)}$ represent the number of shortest paths between nodes j and k , and $\sigma_{(j,k)}(v)$ the number of those paths that traverse node v . The betweenness of v is thus defined as:

$$\beta_v = \sum_{j \neq v \neq k \in \mathcal{V}} \frac{\sigma_{(j,k)}(v)}{\sigma_{(j,k)}}. \quad (2)$$

3. *Closeness* - The closeness γ_v of node v captures how close it is from all other reachable nodes in the network. Given $\pi(v,k)$, the length of the shortest path between v and any other reachable node k , γ_v is defined as:

$$\gamma_v = \left[\sum_{k \neq v, k \in \mathcal{V}} \pi(v,k) \right]^{-1}. \quad (3)$$

As the network evolves over time, we recompute the centrality metrics accordingly for the current snapshot. Thus, in each snapshot \mathcal{G}_t , nodes may have distinct centrality values.

3. DIFFUSION IN DYNAMIC NETWORKS

Suppose a node u needs to send a message to a node v . At the beginning of each snapshot \mathcal{G}_t , each node can store the messages they have or forward those messages to any of its neighbor nodes. A path $u \rightarrow v$ is then set over the dynamic network graph until reaching node v . Note, however, that even if there is no direct path between u and v at any given snapshot, a path may exist between this node pair over the evolving network due to the topology dynamics.

At each snapshot, information can be relayed over a set of nodes in different ways. Basu et al. [4] describe the Store or Advance (SoA) model in which each node can forward a message only to one of its direct neighbors, and that is assumed to take one snapshot. We may use a flooding or a selective forwarding algorithm to spread information over the network. A flooding diffusion process sends information to all reachable nodes in the graph at the same time. For example, in the SoA model, all directly connected neighbors receive the message. In contrast, the selective forwarding algorithm forwards the message to one randomly selected reachable node at each snapshot. The (random) sequence of nodes selected this way is a random walk over the graph [21].

We extend the SoA model with centrality measures (see Section 2). Such an extension induces a *biased* selection of nodes to receive the forwarded information at each step, thereby accelerating diffusion and thus reducing the network cover time. In other words, at each forwarding step, the neighbor node with the highest *degree*, *betweenness*, or *closeness* value is chosen to receive the forwarded message. Hence, a walk over *preferential* nodes is performed. The resulting process is similar to the random walk on a graph, except that next-hop nodes now are chosen based on the highest centrality measures. We thus refer to these diffusion algorithms that take into account centrality measures as *degree walk*, *betweenness walk*, and *closeness walk*, respectively. The intuition behind using centrality measures to drive the next-hop selection in the diffusion process relies on using nodes that present higher relative importance in the network according to some metric as relays to accelerate the information diffusion, whereas with low communication overhead.

We briefly analyze the overhead of the discussed approaches in terms of the amount of information spread over the network to cover it. Two types of information traverse the network: (i) data information, with size τ_d ; and (ii) control information, with size τ_c . Data information can be, for instance, a chunk of video in peer-to-peer live streaming applications, whereas control information carries the node centrality measure. Clearly, $\tau_c \ll \tau_d$.

Considering flooding, the total amount of information spread over the network is proportional to $\mathcal{I}_f = \tau_d |\mathcal{V}|^2$. In the centrality-based algorithms, information exchange is instead divided into two steps. First, the neighborhood of a given node has to send the centrality measure to it. Let us consider, w.l.o.g., that d_m is the mean neighborhood size. The total amount of information in the first step is equal to $\tau_c d_m$. Second, the node with the largest measure is chosen and one message is delivered. The total amount

of information in this second step is τ_d . Consider also that the average path length in the network is π_a . For centrality-based algorithms, the total amount of information spread over the network is $\mathcal{I}_c = \pi_a(\tau_d + \tau_c d_m)$. As typically, for practical large networks, $\pi_a \ll |\mathcal{V}|$, one may expect that applying centrality-driven algorithms generates much less overhead in the network than flooding. Further, the overhead for the centrality-driven algorithms is close to the random walk case.

Finally, we remark that we are not considering here the costs for calculating the centrality measures at each node. We are interested only in analyzing how centrality measures can speed up the diffusion process in complex dynamic networks. Methods for distributively assessing network centrality with low computational and message costs can be found in the recent literature [10, 31].

4. DATASETS

Our experiments use two datasets representing different types of real networks, levels of network dynamics, and topological properties. These datasets are:

1. *Infocom dataset* [27]: This dataset contains the amount of two-day contacts between different pairs of iMotes devices (The iMotes are small sensor platforms with Bluetooth.) distributed to about 70 students and researchers in the Infocom 2006 Conference. In addition to the mobile devices, a number of stationary nodes have been deployed in different floors of the conference hotel. The mobility of attendees has been logged. We consider snapshots with duration $\delta = \{1, 15\}$ in minutes.
2. *SopCast dataset* [30]: The second dataset represents a real P2P network of a live video streaming application. The important characteristic of this dataset is the high level of system dynamics. This dataset includes all the data exchanged among clients (peers) watching a private SopCast channel (<http://www.sopcast.org/>), without any outside interference. Sopcast is a widely popular P2P-TV application, in which peers establish partnerships and exchange data among themselves to receive the live streaming video. We have collected one-hour of exchanging data. We consider $\delta = 1$ second snapshots in order to catch the dynamic behavior of the network. The total number of 334 nodes have their data exchanging process logged.

5. SPEEDING UP A DIFFUSION PROCESS

In this section, we analyze the impact of the network dynamics and the adoption of centrality walks on the diffusion process in dynamic complex networks. In order to compare the performance of the diffusion algorithms, we define η_t as the ratio of reachable nodes at snapshot \mathcal{G}_t :

$$\eta_t = \frac{\sum_{u \in \mathcal{V}_t} r_t(u)}{|\mathcal{V}_t|}, \quad (4)$$

where $r_t(u)$ is the total number of nodes reachable from node u over snapshot \mathcal{G}_t . In other words, to avoid a bias on only considering the result for the best positioned node for diffusion, we take the conservative approach of considering the average case scenario for all nodes in the network.

5.1 On how dynamics impacts network walks

We first investigate how dynamics impacts on the diffusion process using the SopCast dataset. The network cover time for the static \mathcal{G} graph is very fast. Using flooding, 100% of nodes are reached in the second snapshot. Centrality-driven algorithms cover all the network in the third snapshot; a random walk algorithm needs four snapshots in order to accomplish the diffusion process. The cover time over the dynamic $\mathcal{G}_{t=1, \dots, n}$, instead, is much larger. Figure 1 shows the result. Disregarding dynamics underestimates the cover time needed for spreading information over the network in 20 times. In a large set of real systems, it is important to estimate the diffusion time as accurately as possible [32].

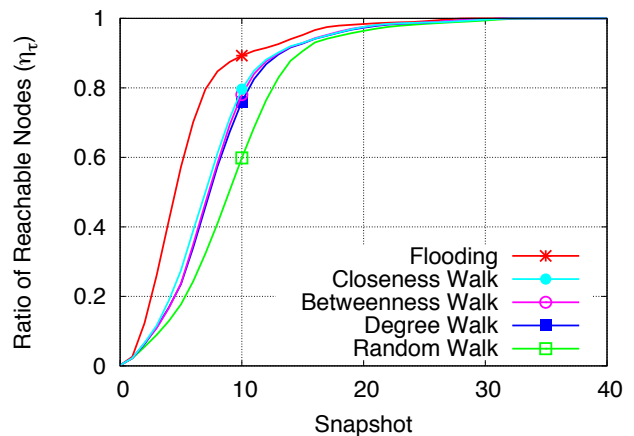


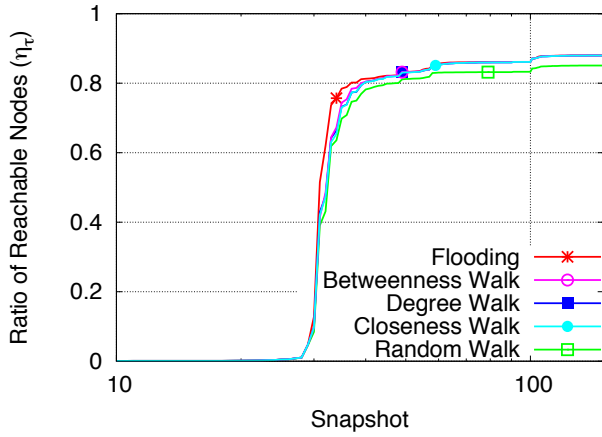
Figure 1: Impact of dynamics on diffusion processes.

We remark that not only the dynamics influences the diffusion time. Figure 1 shows that using centrality measures for forwarding decisions also improves the diffusion process. In the SoA model, using centrality metrics decreases the diffusion time, for the same portion of covered nodes, with respect to using a simple random walk. In other words, choosing the most central nodes accelerates the diffusion process, considering the total time for spreading some information over a portion of nodes. Further, as expected, the best results concerning cover time are reached by the flooding algorithm, but at the price of a very high overhead. Finally, the degree walk performs similarly to the more costly betweenness and closeness walks, reinforcing the idea of adopting the centrality measure with the lowest computational complexity [29].

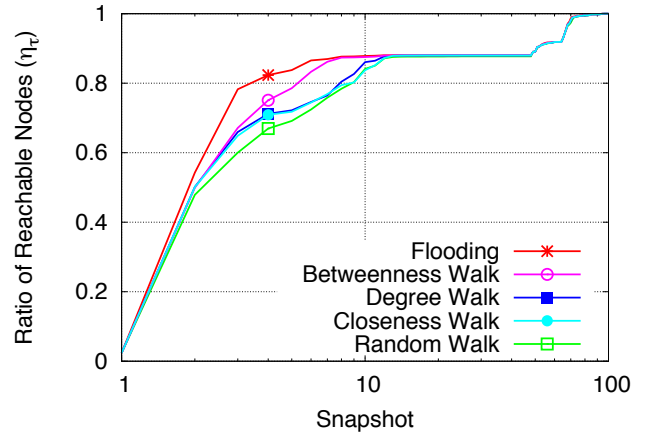
5.2 System knowledge and network walks

As centrality measures accelerate diffusion process, it is important also to analyze how system knowledge contributes for improving the diffusion itself. Here, system knowledge is proportional to the snapshot size δ we use for modeling the set of graphs that represents the system dynamics.

For the Infocom dataset, Figures 2(a) and 2(b) show that choosing snapshots with $\delta = 15$ instead of snapshots with $\delta = 1$ accelerates the diffusion process as well as increases the total number of users that receives the message from 85% to 100%. Interestingly, for the scenario with $\delta = 15$ snapshots, the betweenness walk approximates better the



(a) Using $\delta = 1$ snapshots



(b) Using $\delta = 15$ snapshots

Figure 2: Impact of the system knowledge (δ) on the ratio of reachable nodes.

flooding algorithm and spreads faster the information when compared with the other centrality walks.

As the adopted dataset (Infocom) represents the contacts among conference participants, it is important to know how the logged users behave. In this particular system, higher system knowledge (i.e., higher δ) helps on choosing the key participants with the largest betweenness values. These participants act as bridges connecting different community/clusters in the network and can be interpreted as the most popular participants who know a large number of people. Besides, they tend to have high mobility, reaching 100% of logged conference participants.

6. LINK PREDICTION

If nodes (peers, participants, and so on depending on the modeled network) could predict that a given convenient link is likely to appear soon, they may take the risk of waiting until that link appears instead of forwarding the message at once. For instance, in the illustrative scenario presented in Figure 3, a given node v must send a data to node u . During the initial period ($t = 0$), node v has no option other than to forward data to node a hoping that the data will eventually reach node v at some point in the future. Nevertheless, if node v could predict that the link $v \rightarrow u$ will exist in the very near future (e.g. during $t = 1$ in Figure 3(b)), node v may then choose to wait a while and deliver the data directly to node u . If node v is unable to predict the existence of the link $v \rightarrow u$ in the near future, data dissemination to node u will last for 3 time units (considering that each data hop takes one time unit). In contrast, if node v predicts the existence of the link $v \rightarrow u$, data dissemination will last for 2 time units. If node v fails in its prediction, data dissemination will be penalized. In this case, node v would wait 4 time units until the data reaches node u .

Based on this simple example, it is clear that a link prediction mechanism must be both simple enough to be cost effective and accurate enough to make the miss rate as low as possible. We here propose a link prediction mechanism based on a 2-state Markov model, in which a given link exists in state E and does not exist in state $!E$.

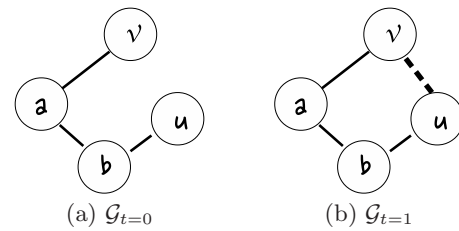


Figure 3: Two consecutive snapshots illustrating the appearance of a link.

Figure 4 illustrates our simple link prediction mechanism. According to this model, the link $v \rightarrow u$ may exist or not during a given snapshot. At time t , if the link exists, it does not exist at time $t + 1$ with probability equal to λ and it keeps existing at time $t + 1$ with probability equals to $(1 - \lambda)$. Similarly, if the link does not exist at time t , it exists at time $t + 1$ with probability equals to ρ and remains inexistent with probability $(1 - \rho)$.

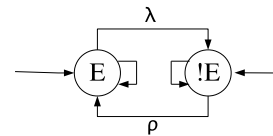


Figure 4: Model for link state dynamics.

To have a temporal view of the state evolution of each link, we define $E_{v \rightarrow u}^t$ as the existence of link $v \rightarrow u$ at time t . We make $E_{v \rightarrow u}^t = 1$ if the link $v \rightarrow u$ exists at time t and $E_{v \rightarrow u}^t = 0$, otherwise. Let $E_{v \rightarrow u}^{t+1}$ represent the existence of the link $v \rightarrow u$ in the next snapshot \mathcal{G}_{t+1} . We may predict $E_{v \rightarrow u}^{t+1}$ as a moving average of the past b sampled values of the link $v \rightarrow u$. Each past value of the link state ($E_{v \rightarrow u}^t, E_{v \rightarrow u}^{t-1}, \dots, E_{v \rightarrow u}^{t-b}$) is weighted by a factor

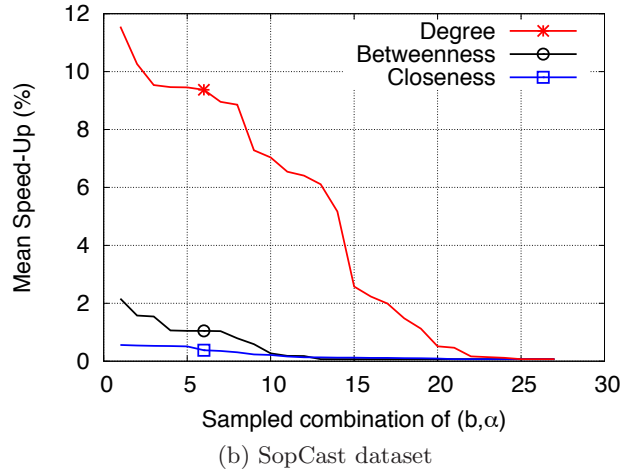
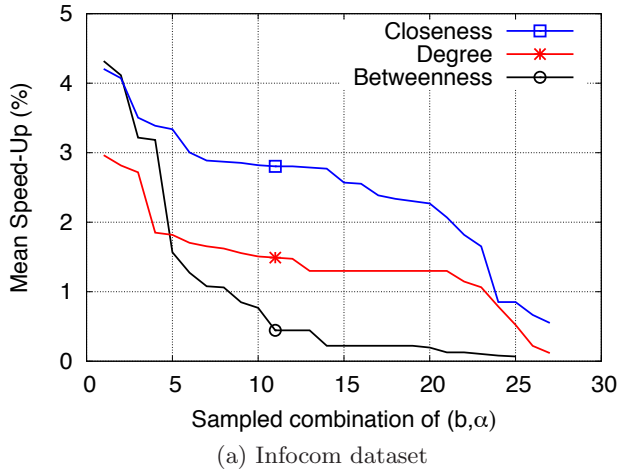


Figure 5: Data dissemination speed-up using link prediction.

α that decreases exponentially. In this way, Equation 5 shows the predicted probability of $E_{v \rightarrow u}^{t+1}$:

$$P(E_{v \rightarrow u}^{t+1}) = \frac{E_{v \rightarrow u}^t + E_{v \rightarrow u}^{t-1} * \alpha + E_{v \rightarrow u}^{t-2} * \alpha^2 + \dots + E_{v \rightarrow u}^{t-b} * \alpha^b}{1 + \alpha + \alpha^2 + \dots + \alpha^b}, \quad (5)$$

where α ($0 < \alpha < 1$) is a decreasing factor. In this sense, older sampled values quickly have their importance reduced in Equation 5. From this sequence of b sampled values, we can also parametrize λ and ρ in the model of Figure 4.

Clearly, the joint choice of values for b and α influences the behavior of the link prediction. We then perform a first study on considering the combination of different values for b and α ranging from 5 to 50 and 0.1 to 0.9, respectively. Figures 5(a) and 5(b) show the impact of link prediction on the centrality walks for diffusion in the Infocom and Sopcast datasets, respectively. In these figures, we show the mean speed up compared with the values found using the same centrality walk without link prediction for different combined samples of (b, α) in the aforementioned ranges. These combined samples (b, α) are ordered in the x-axis in decreasing outcomes of mean speed up. For both datasets, we clearly note that link prediction accelerates the diffusion process. For instance, using the betweenness walk, the data diffusion speed up reaches a factor of 4.3%. For the SopCast dataset, even if it is a random-like graph, we observe a speed up factor up to around 11.6% when we use the degree walk.

These preliminary results encourage further investigation on link prediction as an accelerating factor for centrality-driven diffusion processes, including possible techniques or heuristics to determine convenient combinations of b and α for particular networks. This is a venue we intend to explore in future work.

7. RELATED WORK

There is a large number of previous works that investigate searchability in networks [1, 5, 7, 15, 22, 26]. In this context, searchability is the process of sending a message from a source node to a given destination node in

the network. Most of these works exploit local information intending to enhance the data delivery process. In other words, nodes do not randomly select a neighbor node to forward the data that must reach a given end node. Nodes choose a neighbor node using some metric, expecting that the selected node will be closer to the end node.

For example, Lukose et al. [22] exploit local information about the node neighborhood to propose a heuristic to enhance searchability in power-law networks. Their search strategies use nodes with higher degrees to make the data delivery process faster (i.e., close to the shortest path between the source and the destination). To achieve this, at each step, each node chooses the highest degree node among its neighbors to receive the message to be delivered. Kim et al. [15] use a similar approach to select the next hop of the data delivery process. They compare three heuristics for that: a random choice, a deterministic node choice using the maximum degree among the neighbors, and a probabilistic choice favoring the selection of neighbor nodes with higher connectivity.

Rosvall et al. [26] also investigate the searchability in networks. However, they define the searchability of networks in terms of the difficulty of sending a signal between two nodes in a network without disturbing the remaining network. Authors show that scale-free networks are relatively difficult to search, considering the necessary information to walk the shortest path from a starting point to an end point. Moreover, authors show that real-world networks with higher order organization, like a modular or hierarchical structure, are even more difficult to navigate than random scale-free networks [26]. Despite the importance of investigating the interplay between searchability of a network and its structure, authors disregard the network dynamics. As we show in the present work, the cover time of the information diffusion process may be highly underestimated if network dynamics is disregarded.

Several types of local information have been used intending to enhance message delivery. For instance, Adamic and Adar [1] investigate the selection of the next step at each hop as the neighbor node with the best

connectivity or the neighbor node which is the closest to the target in an organizational hierarchy. In this latter case, the metric calculation may be a problem to the system, as it is unclear how such a metric evolves over time and, in some cases, the determination of physical proximity may be unfeasible. In turn, Şimşek and Jensen [7] show that a simple product of the degree and homophily measures can be quite effective in guiding local search. According to their work, the more similar a neighbor node is to the target and the higher the degree of this neighbor node, the larger the probability for this neighbor node to be close to the shortest path towards the target. In fact, the data delivery process can be close correlated to the network structure and its relationships formation, thereby hidden metric spaces, underlying real networks, may conduct to better delivery heuristics [5]. Therefore, understanding the fundamental laws describing relationships between structure and function of complex networks is key in this area.

We point out that all these works highlight that a central challenge in complex networks is directing messages to specific nodes through a sequence of local decisions made by individual nodes without global knowledge of the network. Most of them rely (at least, partially) on the node degree to take a local decision on the most suitable next hop for the message forwarding among the neighbor nodes at each step of the diffusion process.

Despite the importance of the problem, none of these works discussed so far address an evolving network environment. All network models present a static snapshot of the complex network. As we show in the present work, the cover time of the information diffusion process may be highly underestimated if we disregard that the network evolves. Network topology may change in a very short period. As a consequence, all key local metrics used to determine the next hop, such as the node degree, may also change in a very short period. Therefore, given the importance of taking into account network dynamics, there is an ever-increasing interest in considering time-varying graphs that are able to represent dynamic networks [13, 19, 20]. In particular, there are several studies investigating how key properties of the network, such as node connectivity and centrality, behave in time-varying graphs [12, 16, 25, 28]. Our present work follows this trend further investigating the centrality-driven diffusion in complex dynamic networks.

There is also a number of works that investigate the information diffusion process in complex networks. For example, in [14], authors show how to speed up information diffusion by applying two concepts from complex networks: community structure and popularity. Nodes are split onto different communities and, for each community, nodes with the greatest popularity are chosen for receiving information. Popularity is measured by means of the betweenness metric. In contrast, we are here interested on analyzing the impact of using different centrality measures, not only betweenness, for the acceleration of the diffusion process.

Social relationships are also explored in [23] for information forwarding. Authors define a new metric, named people rank, in order to rank nodes in decreasing importance within the network. The proposed measure, somewhat similar to the page rank metric [6], requires the

knowledge of further information, as social relationships, which is not always available, in order to improve the diffusion. In our study, we need only topological characteristics in order to rank the most important nodes.

Finally, authors in [3] also need social information from network nodes in order to rank the *very important* (VIP) nodes. These nodes, with their movements and interactions, are able to communicate with all the remaining of the network. Similar to [14], nodes are divided into two subsets: global *VIPs* and local *VIPs*. Although the approach we investigate here is also concerned to nodes that act as bridges, the diffusion process we consider does not need any kind of layered organization.

8. CONCLUSION

In this paper, we investigate the adoption of centrality-based metrics in diffusion processes in complex dynamic networks. We show that, if network dynamics is disregarded, a typical model for diffusion process may significantly underestimate the network cover time. Moreover, we show that a centrality-driven selection of the next-hop for the information forwarding can accelerate the diffusion process in dynamic networks with a relatively low message cost. Such a biased selection based on centrality metrics levels off a trade-off between a low cover time with high message cost provided by flooding and large cover time with low message cost provided by a simple random walk. Additionally, we also bring some first results on how to speed up the diffusion process, and thereby reduce the network cover time, by adopting a link prediction scheme in complex dynamic networks. We believe these preliminary results encourage further investigation on link prediction as an accelerating factor for centrality-driven diffusion processes in dynamic networks. This is our target for future work, including the research on possible techniques or heuristics to determine convenient combinations of the involved parameters for particular networks.

9. ACKNOWLEDGMENTS

This work was partially supported by the Brazilian Agencies FAPEMIG, FAPERJ, CAPES, and CNPq.

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